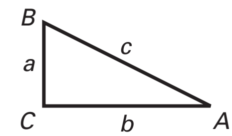
4.5 The Converse of the Pythagorean Theorem

Goal Use the converse of the Pythagorean Theorem. Use side lengths to classify triangles.

THEOREM 4.8: CONVERSE OF THE PYTHAGOREAN THEOREM

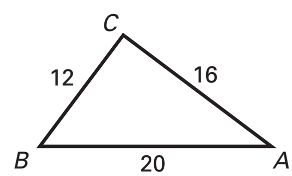
Words If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



Symbols If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Example 1 *Verify a Right Triangle*

Is $\triangle ABC$ a right triangle?



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$c^2 \stackrel{?}{=} a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$(20)^2 \stackrel{?}{=} (12)^2 + (16)^2$	**Substitute values for a, b, and c.**
$400 \stackrel{?}{=} 144 + 256$	**Multiply.**
$400 = 400$	**Simplify.**

Answer It is true that $c^2 = a^2 + b^2$. So, $\triangle ABC$ is a right triangle.

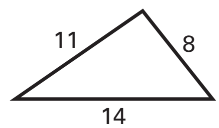
Follow-Up

In Example 1, how do you know which side lengths to use for *a*, *b*, and *c*?

The longest side length is *c* and the other two side lengths are *a* and *b*.

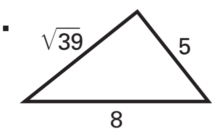
✔ *Checkpoint* Is the triangle a right triangle? Explain your reasoning.

1.



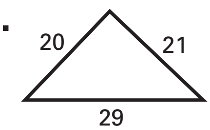
No;
$14^2 \stackrel{?}{=} 8^2 + 11^2$
$196 \neq 185$

2.



Yes;
$8^2 \stackrel{?}{=} (\sqrt{39})^2 + 5^2$
$64 = 64$

3.

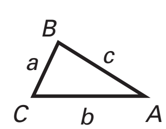


Yes;
$29^2 \stackrel{?}{=} 20^2 + 21^2$
$841 = 841$

CLASSIFYING TRIANGLES

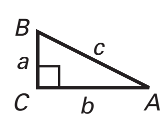
In $\triangle ABC$ with the longest side c:

If $c^2 < a^2 + b^2$,



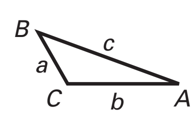
then $\triangle ABC$ is acute.

If $c^2 = a^2 + b^2$,



then $\triangle ABC$ is right.

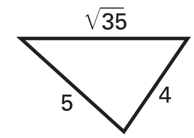
If $c^2 > a^2 + b^2$,



then $\triangle ABC$ is obtuse.

Example 2 *Acute Triangles*

Show that the triangle is an acute triangle.



Solution

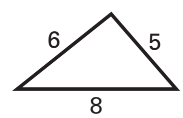
Compare the side lengths.

$c^2 \stackrel{?}{=} a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$(\underline{\sqrt{35}})^2 \stackrel{?}{=} (\underline{4})^2 + (\underline{5})^2$	**Substitute for *a*, *b*, and *c*.**
$\underline{35} \stackrel{?}{=} \underline{16} + \underline{25}$	**Multiply.**
$\underline{35}\ \underline{<}\ \underline{41}$	**Simplify.**

Answer Because c^2 $\underline{<}$ $a^2 + b^2$, the triangle is $\underline{\text{acute}}$.

Example 3 *Classify Triangles*

Classify the triangle as *acute*, *right*, or *obtuse*.



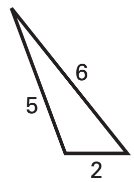
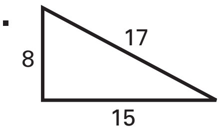
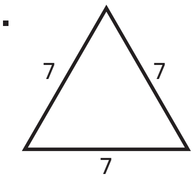
Solution

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

$c^2 \stackrel{?}{=} a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$(\underline{8})^2 \stackrel{?}{=} (\underline{5})^2 + (\underline{6})^2$	**Substitute for *a*, *b*, and *c*.**
$\underline{64} \stackrel{?}{=} \underline{25} + \underline{36}$	**Multiply.**
$\underline{64}\ \underline{>}\ \underline{61}$	**Simplify.**

Answer Because c^2 $\underline{>}$ $a^2 + b^2$, the triangle is $\underline{\text{obtuse}}$.

✔ *Checkpoint* **Classify the triangle as *acute*, *right*, or *obtuse*. Explain.**

4.  obtuse; $6^2 \stackrel{?}{=} 5^2 + 2^2$ $36 > 29$	**5.**  right; $17^2 \stackrel{?}{=} 8^2 + 15^2$ $289 = 289$	**6.**  acute; $7^2 \stackrel{?}{=} 7^2 + 7^2$ $49 < 98$

Use the side lengths to classify the triangle as *acute*, *right*, or *obtuse*.

7. 7, 24, 24 acute	**8.** 7, 24, 25 right	**9.** 7, 24, 26 obtuse